Application of Eigenvalues and Eigenvectors in Facial Expression Recognition

Shanice Feodora Tjahjono 13523097[1,2]
Program Studi Teknik Informatika
Sekolah Teknik Elektro dan Informatika
Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia
[1]13523097@mahasiswa.itb.ac.id, [2]xianfeodora@gmail.com

***Abstract*—No matter where you are in the world, facial expressions are essential as they may express feelings that words cannot convey. This study explores the importance of eigenvalues and eigenvectors in facial expression recognition. Principal Component Analysis (PCA) greatly reduces computational complexity while enabling us to extract and preserve the core of facial features. Through this approach, high-dimensional image data is compacted into smaller representations that help to identify sadness, anger, joy, and other expressions. Experimental experiments show the ability of eigen-based techniques to improve accuracy and computational efficiency in modern recognition systems. This study connects fundamental mathematical principles to practical applications, cementing their role in advancing human-computer interaction and behavioral analysis.**

***Keywords*—eigenvalues, eigenvectors, principal component analysis, facial expression recognition**

I. Introduction

Facial expressions have always been essential as they are a nonverbal language component. Humans can anticipate the mental and physical conditions of others by measuring their emotions, which enables them to accommodate their needs better and make decisions to suit them. Thus, recognizing facial expressions is one of the most basic tasks humans can execute.

Emotions are no longer unquantifiable as scientists have developed methods to analyze them since the nineteenth century [4]. Facial features, such as the corners of the lips, eyes, and nose tip, are essential to identifying separate facial emotions [3].

However, several factors have sparked difficulties during the development of facial expression recognition methods, such as age, lighting conditions, noise, subtlety, and other details. An optimal automatic facial expression recognition system needs to perform well independent of these matters. Fortunately, achievements in related fields of studies have made such systems possible [4].

Facial expression recognition is useful in various applications, including communications, healthcare, lie detection, human-computer interactions, and related topics. User-friendly interfaces, boosting security systems by identifying suspicious activity, increasing customer happiness, and facilitating tailored interactions across several platforms and devices are a few instances that are encountered in daily life.

Fundamental concepts from linear algebra, such as eigenvalues and eigenvectors, have been used to overcome these challenges. The data analysis method known as Principal Component Analysis (PCA) minimizes the dimensionality of datasets into principal components. [1]. It is a method derived from the concept of eigenvalues and eigenvectors commonly used in image processing due to its lack of redundancy, noise reduction, and compact representations [2]. This paper explores how eigenvalues and eigenvectors can be applied to facial expression recognition, as they play an important role in feature extraction and classification.

Figure 1. Facial Expression Recognition
Source: S. Gupta, P. Kumar, and R. K. Tekchandani, "Facial emotion recognition-based real-time learner engagement detection system in online learning context using deep learning models," Multimedia Tools and Applications, vol. 82, no. 9, pp. 11365–11394, Sep. 2022

II. Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors are concepts in linear algebra that describe linear transformations and analyze their effects. Given a square matrix A, a non-zero vector, an eigenvector $\mathbf{v}$, that when transformed by A, is scaled by an eigenvalue, λ, as represented by the equation below:

$$A\mathbf{v} = \lambda\mathbf{v}. \tag{1}$$

This equation shows that the eigenvector ($\mathbf{v}$) represents the direction in the vector space that remains constant under the transformation A, while the eigenvalue (λ) quantifies the magnitude and direction of stretching, compression, or reversing.

To compute eigenvalues, the characteristic equation is derived from the determinant of the matrix $\lambda I - A$, where I is the identity matrix.

$$\det(\lambda I - A) = 0 \tag{2}$$

The solutions to this polynomial equation, $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$, are the eigenvalues of A. When we substitute these eigenvalues back into

$$(\lambda I - A)\mathbf{v} = 0, \tag{3}$$

this yields the corresponding eigenvectors $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_n)$.

Eigenvectors represent the "preferred directions" of the linear transformation A, where the transformation acts purely by scaling without changing the vector's direction. The eigenvalue associated with each eigenvector indicates the extent of this scaling. For example:

1. If $\lambda > 1$, the vector is stretched.
2. If $0 < \lambda < 1$, the vector is compressed.
3. If $\lambda < 0$, the vector is scaled and reversed in direction.

Symmetric matrices, which frequently appear in applications like Principal Component Analysis (PCA), have orthogonal eigenvectors, making them useful for decomposing transformations [5].

In data analysis, eigenvalues and eigenvectors find practical application in the covariance matrix—a mathematical representation of the relationships between variables in a dataset. The covariance matrix serves as the foundation for extracting eigenvalues and eigenvectors, which reveal the principal directions of variance and their magnitudes. By decomposing the covariance matrix, we can identify patterns in the data and uncover its underlying structure, as is done in Principal Component Analysis (PCA).

The pairwise covariance between variables in a dataset is represented by a square matrix known as the covariance matrix. Covariance measures how two variables vary together—whether an increase in one variable corresponds to an increase or decrease in another. Covariance is measured between two dimensions. In datasets with more than two dimensions, multiple covariance values are calculated, one for each pair of dimensions. To obtain these values, the covariance matrix comes into play with the following calculation:

$$C_{MxN} = \left(c_{i,j}\right), where\ c_{i,j} = cov(Dim_i, Dim_j) \tag{4}$$

where C_{MxN} is a square matrix ($M = N$) with M rows and N columns. Dim_x represents the x-th dimension. Each element ($c_{i,j}$) in the matrix represents the covariance between the i-th and j-th dimensions. For instance, the value at row 2 and column 3 corresponds to the covariance between the second and third dimensions of the dataset.

In PCA, the covariance matrix is computed from the dataset to identify the directions (or principal components) of maximum variance. The eigenvectors of the covariance matrix correspond to these directions, and the eigenvalues indicate the amount of variance captured along each direction [1].

III. PRINCIPAL COMPONENT ANALYSIS

Principal Component Analysis (PCA) was first introduced by Karl Pearson in 1901 as a method for fitting lines and planes to data in multi-dimensional space, laying the foundation for modern multivariate statistics. Later, Harold Hotelling extended and formalized PCA in 1933, developing its current formulation to address problems of multivariate data reduction. The technique became widely recognized for extracting patterns and reducing dimensionality in large datasets [1], [6].

Principal Component Analysis (PCA) is a statistical technique used to identify patterns in data and express the data in a way that emphasizes their differences and similarities. This is achieved by transforming the original dataset into a new coordinate system defined by principal components, uncorrelated variables derived from the original correlated ones.

PCA primarily serves as a dimensionality reduction tool, reducing the number of variables in a dataset while keeping the most informative ones. This makes data easier to analyze and operate while solving problems related to compression, recognition, and feature extraction. It involves a numerical procedure that converts the original data into a set of uncorrelated variables called principal components. Then, it arranges these components so that the first component captures the greatest variance in the data, and each subsequent component captures progressively less variance.

Due to its effectiveness, PCA has been applied in various fields, such as face recognition, image denoising, data compression, data mining, and machine learning [7].

IV. Discussion

A. The Role of PCA in Facial Expression Recognition

While facial expression recognition technology has advanced significantly, there are still some issues that need to be resolved, and new obstacles continue to surface, especially when the technology is used in more complicated and varied real-world situations. Some of the main challenges include ambiguous facial expressions, environmental variables, and limited datasets [8].

PCA has become a widely adopted approach to overcome such challenges. It boosts the accuracy and efficiency of facial expression recognition systems by offering a robust framework to manage variations in emotions, lighting, and occlusions. PCA achieves this by simplifying complex facial datasets while retaining the key features necessary for recognition. By analyzing variance, extracting features, and projecting the data into a reduced-dimensional eigenspace known as eigenfaces, PCA identifies patterns in facial data. This process reduces complex facial structures to their core components, enabling efficient and manageable computation [7].

In computer vision, PCA has become a fundamental approach, especially for problems such as face expression recognition. PCA was originally limited to statistical applications, but with the progress of digital image processing, it gained traction for addressing challenges such as feature extraction, noise reduction, and dimensionality reduction. Techniques like Eigenfaces utilize PCA to condense high-dimensional facial data into a more manageable format while retaining the essential features needed for accurate recognition. Its continued importance can be attributed to its adaptability, which combines conventional methods with modern AI to guarantee reliable and efficient recognition in a range of real-world scenarios [9], [10].

B. Generating Eigenfaces

Eigenfaces are a set of eigenvectors taken from the covariance matrix of facial image data. These eigenvectors act as the main components of the face dataset and capture the biggest differences in facial features. The method reduces the size of the data by mapping it onto a smaller space made up of these eigenvectors, called the "eigenface space." Each eigenface represents a specific variation in the dataset, like lighting, pose, or expression. A face can be described as a combination of these eigenfaces with different weights, making recognition easier and faster. This approach cuts down on computational work while keeping the important details needed to tell faces apart. The Eigenfaces method has become a cornerstone of modern face recognition systems, demonstrating the effectiveness of Principal Component Analysis (PCA) in identifying meaningful patterns within high-dimensional facial data [9].

The Eigenface method involves several key steps in its mathematical foundation. It begins with calculating the covariance matrix of the dataset to identify relationships between pixel intensities in facial images. The eigenvalues and eigenvectors of this covariance matrix are then extracted, with the eigenvectors representing the principal components of the data. Using these components, the facial data is projected into a lower-dimensional space—referred to as eigenfaces—which highlights the most significant differences in the images. This process not only simplifies the data but also preserves the critical features necessary for facial recognition. Examples of eigenface generation often use datasets like the Indian, Utrecht, and other custom databases, providing a variety of facial images to support the methodology. The resulting eigenfaces are visualized as principal components [6], [7], [9].

C. Classification of Expressions

The process of expression classification can be divided into two main stages: feature extraction and expression categorization.

Facial feature extraction starts by locating essential points on the face, including the lips, eyes, and nose. These points are crucial for interpreting facial expressions since they are the areas where significant changes happen to communicate emotions. Correctly identifying these points enables the system to concentrate on significant facial areas for analyzing expressions. Research such as "PCA-based Dictionary Construction for Accurate Facial Expression Recognition via Sparse Representation" shows the significance of these aspects for dependable data representation and later processing by pinpointing these essential points, where features are derived from facial images through PCA. Converting high-dimensional image data into lower dimensions (principal components) emphasizes the variations crucial to differentiating various expressions, reducing redundancy, and facilitating efficient data management [11].

Building a strong facial expression recognition (FER) system requires the system to classify the extracted face features into universal emotion categories. This approach is aided by PCA's simplified representation of facial data, which highlights the key aspects of the retrieved features. Papers such as "Facial Expression Recognition Utilizing PCA-based Interface for Wheelchair" demonstrate how PCA helps in expression classification [12].

Expression classification uses a range of methods, from simpler techniques like Euclidean distance to more advanced ones such as Support Vector Machines (SVM) and neural networks. These methods benefit from PCA’s reduced-dimensional data, enabling accurate and efficient classification. An example would be how the paper

"Facial Expression Recognition Based on Fusion Feature of PCA and LBP With SVM" shows how combining PCA with machine learning significantly enhances the speed and accuracy of expression recognition systems [13].

D. Training and Testing the Facial Expression Recognition (FER) Model

We will look at the training and testing methodology in the paper “A Principal Component Analysis of Facial Expressions” [6]. The training phase involves preparing the system with four distinct datasets:

1. Utrecht Database: Comprising 131 images of male and female subjects with smiling and natural expressions, this dataset is used to establish foundational patterns in facial expressions.

2. Indian Database: Contains 50 images of male subjects with diverse emotions, such as happiness, sadness, anger, and neutrality, to capture a range of expressions.

3. Researcher Database: This custom database includes 45 images from three individuals, featuring expressions like surprise, anger, and disgust, taken in controlled environments with varied lighting conditions.

4. Pain Expressions Database: A more complex dataset with 599 images (male and female) categorized across nine different expressions, including pain, ensuring robustness in recognition.

Figure 2. Indian Database
Source: [6]

Figure 3. Researcher Database
Source: [6]

These datasets go through PCA-based transformations to reduce dimensionality while keeping the most significant features. The principal components extracted during this process serve as the basis for recognizing patterns in facial expressions. By training on these diverse datasets, the system learns to distinguish between multiple expressions effectively, even under varying conditions.

Using the four datasets, the facial expression recognition system's testing phase [6] assessed how well it could classify face expressions. With 29 out of 31 photographs properly identified, the Indian Database had a classification accuracy of 93.5%. In the Pain Expressions Database, the accuracy was 99.22% for men and 99.7% for women, for a total accuracy of 98.9%. All 67 photos in the Utrecht Database, which included neutral and joyful facial expressions from both males and females, had a perfect 100% categorization rate. With 44 out of 45 images correctly classified, the Researcher Database, which included photos with a range of facial expressions and lighting conditions, showed a 97.8% accuracy rate. The system's total accuracy was remarkable, accurately detecting 733 out of 742 photos across all databases, returning an astonishing 98.78% overall accuracy.

E. Advantages of PCA in Facial Expression Recognition

Principal Component Analysis (PCA) is an important tool for improving facial expression recognition technology. Its ability to simplify data and improve processing has helped solve many challenges in this field.

PCA works by reducing the size of facial image data while keeping the most important features. This reduction is essential for dealing with large datasets, as it makes calculations easier, speeds up processing, and saves storage space. By finding the key components that show differences in facial expressions, PCA makes analysis and recognition more efficient [2], [9].

PCA also extracts meaningful patterns in facial expression data by focusing on the most important features and reducing the effects of noise or unnecessary details. These features, called eigenfaces, provide a simple yet effective way to classify expressions. This makes recognition systems more accurate and reliable, even in tough conditions such as bad lighting or unreliable backgrounds [3], [7], [9].

By transforming features, PCA reduces multicollinearity and helps models work better, making it a great preprocessing tool for machine learning algorithms like Support Vector Machines (SVMs) and neural networks. This improves how well these models handle facial expression data, making them faster and more effective [10], [13].

Another great advantage of PCA is its ability to handle large datasets. It uses optimized methods to calculate eigenvectors, allowing it to process thousands of high-quality images quickly. This scalability is crucial for real-time applications like recognizing facial expressions in videos [8], [9].

Finally, PCA is very flexible in managing changes in facial expressions caused by things like age, ethnicity, lighting, or emotional intensity. By focusing on the most important features, PCA and its versatility ensures reliable performance in different real-world situations [6], [7].

These advantages showcase PCA's transformative impact on the development of facial expression recognition technology. Its ability to simplify complex datasets, improve system efficiency, and ensure adaptability has established PCA as an essential technique used in facial expression recognition and other applications.

F. Applications of PCA-Based Facial Expression Recognition

Principal Component Analysis (PCA), which turns abstract linear algebra into useful tools for everyday challenges, has seen significant applicability in real-world situations. Based on the concepts of eigenvalues and eigenvectors, PCA simplifies high-dimensional data, which makes it useful for a variety of applications. The way these seemingly abstract mathematical techniques are applied in real-world situations—analyzing small facial movements to identify emotions, enhance user experiences, and even support mental health assessments—is what makes this technology so fascinating. The way theoretical mathematics is seamlessly integrated into applied technology shows how innovative techniques like PCA may be in solving the setbacks and roadblocks of today.

In human-computer interaction (HCI) systems, PCA improves the user experience by allowing devices to recognize and react to users' emotional states. In adaptive user interfaces, for instance, PCA-based algorithms are employed to ensure a more intuitive interaction by allowing gadgets to adjust their behavior according to the user's mood [4], [8].

To monitor patients' emotional states and spot signs of mental health conditions like anxiety or depression, the medical industry uses PCA-based facial expression recognition. Therapeutic applications benefit from it as well, where strategies for treatment can be formulated by analyzing patients' facial expressions. PCA also supports the advancement of assistive technologies, such as wheelchair-accessible user interfaces. For instance, wheelchair control and nonverbal emotional communication have been addressed by PCA-based facial recognition systems [3], [6].

PCA is used in security systems to identify threatening or suspect behavior based on facial expressions in the surveillance and security industry. It enhances situational awareness and reaction times by assisting in the identification of tension or panic in people at airports, public gatherings, or vital infrastructure sites [8], [9].

Businesses can determine consumer satisfaction levels and adjust marketing strategies by using PCA-based algorithms to analyze customers' facial expressions during product interactions. This has worked well as a corporate strategy in focus groups and retail settings [8], [11].

PCA-based systems are also used to monitor students' engagement and emotional responses during online learning. By analyzing expressions, educators can identify moments of confusion or lack of interest and adapt teaching strategies accordingly [7], [8]. One interesting example is the use of classroom cameras in China that scan students' faces to analyze their emotional responses and engagement levels [14]. These systems monitor expressions to detect emotions such as boredom, confusion, or focus, aiming to provide real-time feedback to educators. While such PCA-based technologies have the potential to enhance learning experiences, it is inevitable that it raises concerns about privacy and the ethics of constant surveillance. This stresses the dual-edged nature of such advancements, offering practical benefits with possible, yet significant societal challenges.

V. CONCLUSION

This paper focuses on the application of mathematical concepts to everyday instances. To reduce data and extract features using Principal Component Analysis (PCA), eigenvalues and eigenvectors are crucial for facial expression identification. This study explains how eigenfaces, formed from eigenvectors, have shown themselves to be versatile and computationally efficient in various applications.

Despite its numerous advantages, there are drawbacks to using PCA exclusively, including sensitivity to preprocessing conditions and challenges in identifying subtle expressions. These difficulties highlight how crucial it is to combine PCA with cutting-edge techniques such as deep learning to increase accuracy and dependability. We can also conclude that eigenvalues and eigenvectors will continue to be pivotal for the advancements of real-time emotion detection and decision-making as facial expression recognition systems develop.

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PERNYATAAN

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Shanice Feodora Tjahjono 13523097